Special Relativity as a Facet of Probability Theory? Exploring Probabilistic Analogies in Competitive Games

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Abstract

This paper uncovers a surprising connection between special relativity and probability theory through simple competitive games. By modeling contests—first as draws from an urn and then as races between exponentially distributed events—we derive expectation values that mirror the relativistic velocity addition formula. Remarkably, these probabilistic outcomes obey Einstein's rule for combining velocities, suggesting a structural analogy between relativistic kinematics and probabilistic competition. This finding blurs the traditional divide between the deterministic framework of relativity and the stochastic nature of quantum mechanics, hinting that probabilistic principles may underpin aspects of special relativity. We provide rigorous derivations and explore implications for interdisciplinary studies in physics and probability.

1 Introduction

Special relativity and quantum mechanics are often viewed as contrasting pillars of modern physics: the former deterministic, governed by the invariant speed of light, and the latter probabilistic, defined by uncertainty. Yet, could a probabilistic framework secretly scaffold the kinematics of special relativity? This paper investigates this possibility by examining competitive games where probabilistic outcomes mimic relativistic velocity addition, as introduced by Einstein [1].

We explore two models: a game of colored balls drawn from an urn and a race between entities with exponentially distributed waiting times. In both, the expectation of victory takes a form analogous to relative velocities, composing according to Einstein's formula when normalized by $c = 1$. This striking parallel suggests that special relativity's mathematical structure might emerge from probabilistic systems, challenging its deterministic label and inviting a rethinking of its foundations.

By connecting a cornerstone of physics to elementary probability, this work offers a novel, accessible analogy. We proceed with detailed derivations and conclude with implications for bridging physics and probability.

2 Model 1: The Urn Game

Consider three players—R, G, and B—with N_R, N_G, and N_B colored balls, respectively. In a match (e.g., R vs. G), their balls are pooled into an urn, and one is drawn randomly. The winner, whose color is drawn, earns +1; the loser gets −1.

2.1 Probability and Expectation

For R vs. G, the total balls are $N_R + N_G$. The probabilities are:

$$P_R = \frac{N_R}{N_R + N_G}, \tag{1}$$

$$P_G = \frac{N_G}{N_R + N_G}. \tag{2}$$

R's expected gain, v_{RG}, is:

$$\begin{aligned} v_{RG} &= P_R \cdot 1 + P_G \cdot (-1) \\ &= \frac{N_R}{N_R + N_G} - \frac{N_G}{N_R + N_G} \\ &= \frac{N_R - N_G}{N_R + N_G}. \end{aligned} \tag{3}$$

Similarly:

$$v_{GR} = \frac{N_G - N_R}{N_R + N_G} = -v_{RG}, \tag{4}$$

$$v_{GB} = \frac{N_G - N_B}{N_G + N_B}, \tag{5}$$

$$v_{RB} = \frac{N_R - N_B}{N_R + N_B}. \tag{6}$$

2.2 Relativistic Composition

Treating v_{RG}, v_{GB}, and v_{RB} as velocities (with $c = 1$), we test Einstein's formula:

$$v_{RB} \stackrel{?}{=} \frac{v_{RG} + v_{GB}}{1 + v_{RG}v_{GB}}. \tag{7}$$

Substituting the expressions from equations (3), (5), and (6), we compute the components step-by-step.

The numerator is:

$$v_{RG} + v_{GB} = \frac{N_R - N_G}{N_R + N_G} + \frac{N_G - N_B}{N_G + N_B} = \frac{2N_G(N_R - N_B)}{(N_R + N_G)(N_G + N_B)}. \tag{8}$$

The denominator is:

$$1 + v_{RG}v_{GB} = 1 + \left(\frac{N_R - N_G}{N_R + N_G}\right)\left(\frac{N_G - N_B}{N_G + N_B}\right) = \frac{2N_G(N_R + N_B)}{(N_R + N_G)(N_G + N_B)}. \tag{9}$$

The result follows as:

$$\frac{v_{RG} + v_{GB}}{1 + v_{RG}v_{GB}} = \frac{2N_G(N_R - N_B)}{2N_G(N_R + N_B)} = \frac{N_R - N_B}{N_R + N_B} = v_{RB}. \tag{10}$$

The equality holds, mirroring relativistic kinematics.

3 Model 2: The Exponential Race

Now, consider devices R, G, and B ringing at rates λ_R, λ_G, and λ_B (exponential distributions). The winner is the first to ring.

3.1 Probability and Expectation

For R vs. G, with $T_R \sim \text{Exp}(\lambda_R)$ and $T_G \sim \text{Exp}(\lambda_G)$, the probability R rings first is:

$$\begin{aligned} P(T_R < T_G) &= \int_0^\infty \lambda_R e^{-\lambda_R t} e^{-\lambda_G t}\, dt \\ &= \frac{\lambda_R}{\lambda_R + \lambda_G}, \end{aligned} \tag{11}$$

and $P(T_G < T_R) = \frac{\lambda_G}{\lambda_R+\lambda_G}$. Thus:

$$\begin{aligned} v_{RG} &= \frac{\lambda_R}{\lambda_R + \lambda_G} \cdot 1 + \frac{\lambda_G}{\lambda_R + \lambda_G} \cdot (-1) \\ &= \frac{\lambda_R - \lambda_G}{\lambda_R + \lambda_G}. \end{aligned} \tag{12}$$

Similarly:

$$v_{GB} = \frac{\lambda_G - \lambda_B}{\lambda_G + \lambda_B}, \tag{13}$$

$$v_{RB} = \frac{\lambda_R - \lambda_B}{\lambda_R + \lambda_B}. \tag{14}$$

3.2 Relativistic Composition

The formula $v_{RB} = \frac{v_{RG}+v_{GB}}{1+v_{RG}v_{GB}}$ holds, as the structure mirrors the urn model with λ replacing N.

4 Discussion

Both models produce expectations of the form $\frac{a-b}{a+b}$, which compose relativistically. The urn game uses discrete probabilities, the race continuous stochastic processes—yet both align with Einstein's velocity addition [1]. This suggests relativity's kinematic structure may emerge from probabilistic competition, challenging its deterministic framing and hinting at deeper stochastic roots.

5 Conclusion

We have shown that probabilistic games can replicate special relativity's velocity addition, offering a novel lens on its foundations. Future work could investigate whether other relativistic effects, like time dilation, arise from similar frameworks, further uniting physics and probability.

References

[1] A. Einstein, "Zur Elektrodynamik bewegter Körper" ("On the Electrodynamics of Moving Bodies"), *Annalen der Physik*, vol. 322, no. 10, pp. 891–921, 1905.